HIERARCHICAL REASONING IN TIME AND SPACE

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ABSTRACT. The study of spatial relationships has been one of the most active areas in GIS research over the past twenty years. Many formal models have been proposed for the definition of topological or cardinal relationships. However, these approaches often consider space as a static continuum which does not integrate the temporal dimension. Recent progress in spatio-temporal GIS research attempts to extend the representation of spatial relationships to the integration of the life and motion of spatial entities or in other words, the study of the evolution of spatial entities. However, these models generally consider the evolution of spatial entities, that is, entities constrained by a filiation tree. This paper proposes an alternative view of relationships in space and time, that is, we consider independent entities in space and time. The temporal and spatial dimensions are modelled using a hierarchical approach that allows the description of relationships at different levels of abstraction. We show that hierarchical reasoning in time and space supports the identification of modular relationships, generally not identified in existing temporal GIS models, and suggest that their use has promising potential for many GIS applications.

1. INTRODUCTION

One of the most important functional properties of GIS is its capability to support the exploration of relationships and patterns in space and time. These include the analysis of real-world phenomena such as periodic spatio-temporal events and processes. Understanding spatio-temporal phenomena is of fundamental interest for the derivation of trends, rules and eventually laws that model events, changes and evolution, to test hypotheses and to develop a better comprehension of real-world dynamics. To date, many GIS database applications have been successfully developed, particularly in environmental and urban areas (Goodchild *et al.* 1996). These applications now provide a rich quantity of data which could potentially offer support to the development of efficient spatio-temporal analysis. However, there is still a need for the integration of the temporal dimension within GIS in order to support the analysis of spatio-temporal phenomena (Langran 1992, Peuquet 1994). Recent advances include the developments of temporal GIS models oriented towards the identification of semantic models for the description of life and motion of spatial entities (Claramunt and Thériault 1995 and 1996, Hornsby and Egenhofer 1997, Thériault *et al.* 1999), and the integration of qualitative temporal reasoning within GIS (Frank 1994) to mention some examples.

In a related work, we propose a model for the description of relationships in space and time, that is, an algebra which identifies spatio-temporal relationships between independent entities (Claramunt and Jiang 2000). In fact, the modelling of relationships that integrate both the temporal and spatial dimensions is still a challenge for GIS research. A closer investigation of the temporal dimension has shown that hierarchical analysis of events and states in time is often employed in many GIS application areas (Whigham 1993). Such a hierarchical representation is based on the fact that events are often referred and organised using a calendar (e.g., year-month-day) that reflects the cyclic structure of time. A calendar introduces a different range of relationships than those usually identified within linear-time algebra: for example two daily events, disjoint on a linear time-line can be closely related within a cyclic representation of time (e.g., July 14, 1998 and July 14, 1999). Therefore, we believe that the analysis of temporal relationships between events are not limited to a strict application of basic operations between temporal intervals in a linear time-line, but instead to an analysis of the "proximity" of these temporal intervals at different levels of the underlying calendar hierarchy that represents the cycle of time. Similarly, the study of spatial relationships can be developed using a nested hierarchical – but not cyclic - organisation of regions in space (Kainz *et al.* 1993, Car and Frank 1994, Kuipers 1996). For example, two disjoint buildings may or may not belong to the same neighbourhood at a higher level of abstraction.

Our model proposes a representation of temporal and topological relationships using a hierarchical and cyclic model of the time-line and a hierarchical model of the two-dimensional space. We show that such a hierarchical approach provides a flexible set of temporal and topological relationships that together provide some nice properties for many GIS applications. The concepts presented in this research are illustrated in the context of a fictive crime study. The remainder of this paper is organised as follows. Section 2 introduces the principles of our model. Section 3 develops the formal representation of hierarchical temporal relationships. Section 4 applies similar hierarchical concepts to spatial reasoning. Finally section 5 concludes the paper.

2. MODEL PRINCIPLES

Current temporal reasoning algebras are based, to the best of our knowledge, on a linear representation of time (Allen 1984). Time is represented as a set of measured times isomorphic to a set of real numbers. As in Allen's algebra, we consider convex periods as the basic temporal primitive of our model. We define a period as an anchored duration of time delimited by two time-stamps, an interval as the length of a period (Snodgrass *et al.* 1995). Allen's algebra defines 7 temporal relationships (13 with their inverses) between temporal periods (Figure 1).

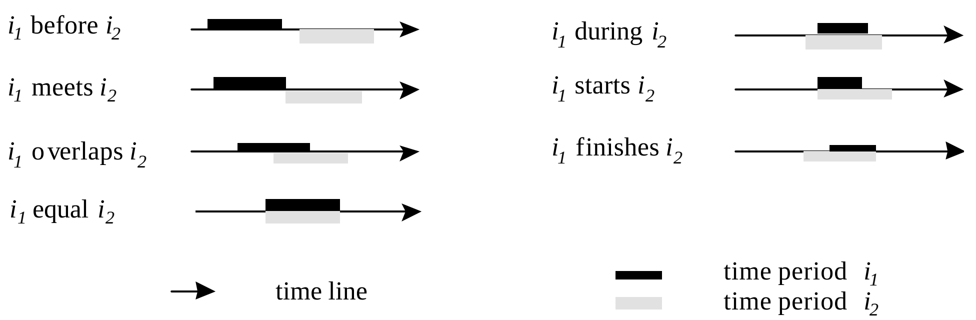

Figure 1: Allen's temporal relationships

This algebra is widely used in temporal reasoning. It has also been considered as a reference, or at least as an equivalent, by temporal database models and query languages (Snodgrass *et al.* 1995). However, we believe that a linear view of the time-line is not sufficient for an analysis of event patterns in space and time. Many application domains widely use a cyclic and hierarchical representation of the temporal dimension. Such applications are based on the concept of calendar, that is, a hierarchical subdivision of time (Figure 2). A calendar is defined by a hierarchy of non-intersecting temporal intervals. Each temporal interval of the hierarchy can be decomposed by a sum of temporal intervals defined at the immediate lower level of the hierarchy, if any (i.e., does not apply for the lowest level of the hierarchy). We denote a calendar as follows:

- Let $C(A_1, A_2, \ldots, A_m)$ be a calendar where $A_1, A_2, \ldots, A_m$ represent the temporal intervals of the calendar hierarchy and *m* the number of levels of the calendar hierarchy, also called the depth of the calendar (Chandra *et al.* 1998).

In general, there are two alternatives to the above notation: either A_1 represents the finest temporal interval level of the hierarchy and A_m the coarsest temporal interval level of the hierarchy, or in contrast, A_1 represents the coarsest temporal interval level of the hierarchy and A_m the finest temporal interval level of the hierarchy. Most Western calendars adopt the former approach, whilst most Eastern calendars take the latter approach. We will take the second option in the remainder of this paper. Let us consider the example of a calendar defined with years, months and days. A year is decomposed in 12 months, a month in either 28, 29, 30 or 31 days (weeks lead generally to some problems in the representation of calendars as the same week can belong to two different months or even years in some cases). The notion of granularity is closely related to the concept of calendar. Granularity is one of the key concepts in temporal database modelling. The granularity of an event can be defined as the smaller temporal interval unit used in the modelling of this event. For instance, the concepts of being early, being late and being at the same time, are strongly affected by the level of granularity used. The required precision of the time-axis depends upon the application. For very dynamic phenomena, minute and second-based granularity units are often used, whilst dealing with more conventional applications, a calendar time scale based upon monthly, weekly or daily granularity units is more relevant.

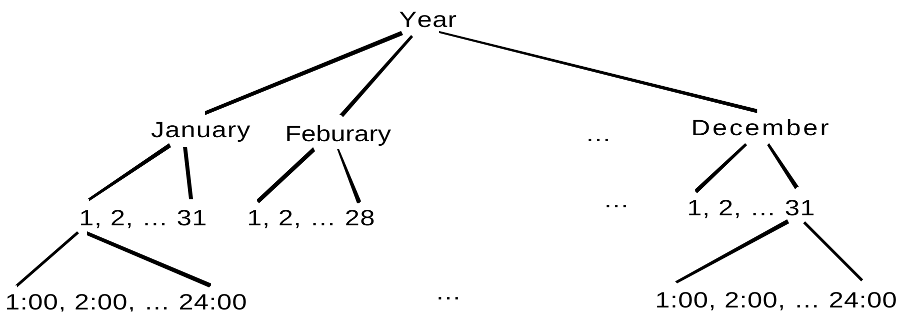

Figure 2: Example of temporal hierarchy

Although some events may be instantaneous, most occur over a period of time (Allen 1991). Accordingly, we give a temporal period extension to an event as this corresponds more closely to real world scenario. An event can be defined as an application-driven concept that supports a cognitive interpretation of a significant pattern of change. The concepts of states and events are often related as discussed in Allen (1991). Our model is oriented to the manipulation of events, it also applies to states as far as these are represented using convex temporal intervals and a calendar. We define an event as follows:

- Let $e_1(a_1, a_2, \ldots, a_p)$ be an event defined using a calendar $C(A_1, A_2, \ldots, A_p, ..., A_m)$ where $a_1, a_2, \ldots, a_p$ represent temporal periods (limited to convex periods within the scope of our model) defined using the temporal intervals of the calendar hierarchy, that is, $A_1, A_2, \ldots, A_p$, respectively ; p is the number of temporal period levels of e_1, $1 \leq p \leq m$, $a_p \subseteq a_{p-1} \subseteq \ldots \subseteq a_1$. A_p gives the granularity of e_1.

Our model does not attempt to provide a general theory for hierarchical reasoning in time. We limit the scope of our framework to short events whose each temporal period is "contained" (i.e., either *during*, *starts*, *finishes* or *equal*) in a temporal period defined by an interval unit of the immediate superior level of the calendar hierarchy (i.e., nested hierarchy). Let us illustrate this concept with a calendar C_1(*Month*, *Day*, *Hour*). For example, an event defined by a granularity at the finest level of this calendar (i.e., *Hour*) is "contained" in a period unit defined at the immediate superior level of the calendar hierarchy, that is, a day period. Therefore, temporal intervals represented at the finest level of such a calendar are restricted to a 24 hour period of reference (e.g., [10:00, 18:00]). Such a restriction applies very well to the study of daily events generally used for the analysis of crime patterns, monitoring of urban traffic conditions or some environmental studies, it is also required to reflect the properties of the nested hierarchy.

Hierarchical reasoning is not restricted to the temporal dimension, space can also be formalised and analysed using a hierarchical view (Hirtle and Jonides 1985, Whigham 1993, Kainz *et al.* 1993, Car 1994, Kuipers 1996). We believe that such a principle is adapted to many domains in which possible relationships between located events involve both temporal and topological relationships. We consider daily events located in space. We model space as a nested hierarchy of non-intersecting regions. Each region of the hierarchy can be recursively decomposed (i.e., dis-aggregated) by several regions of space at the immediate lower level – if any – of that hierarchy. Using such a hierarchy, each event in space can be recursively decomposed into several regions of space. Administrative and political subdivisions are often organised using such a hierarchy. Let us take the example of an urban environment, that is, a city composed of a set of non-intersecting districts, each district composed of a set of non-intersecting neighbourhoods, and each neighbourhood composed of a set of non-intersecting buildings. Such a spatial hierarchy can be illustrated graphically as in Figure 3.

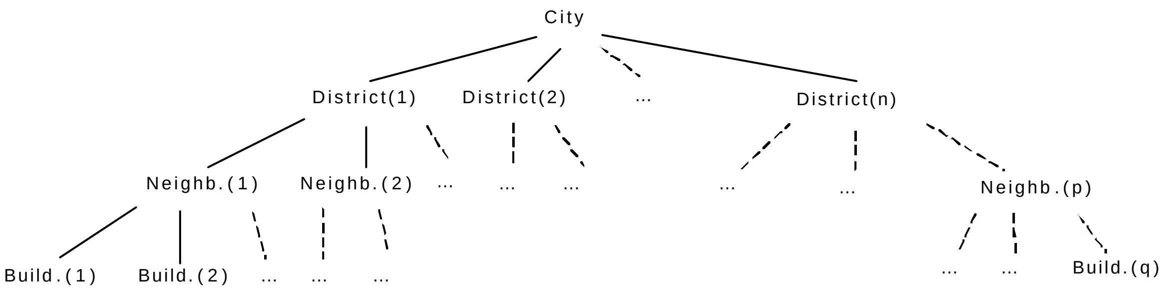

Figure 3: Example of spatial hierarchy

We define a spatial hierarchy as follows:

- Let $SH(S_1, S_2, \ldots, S_n)$ be a spatial hierarchy where $S_1, S_2, \ldots, S_n$ represent successive levels of abstraction in the spatial dimension, from the highest, that is S_1, to the lowest, that is S_n, levels of abstraction, and n the number of levels of abstraction of *SH*.

Then let us define an event located in space:

- Let $e_1(a_1, a_2, \ldots, a_p)$ be an event defined using a calendar $C(A_1, A_2, \ldots, A_p, ..., A_m)$. We say that e_1 is – hierarchically – located in space iff there is a spatial hierarchy $SH(S_1, S_2, \ldots, S_q, \ldots, S_n)$ such as e_{s1} $(s_1, s_2, \ldots, s_q)$ represents the nested regions of SH that recursively locate e_1 in space, $1 \le q \le n$, s_1 (*equal* $\vee$ *covers* $\vee$ *contains*) $s_2, \ldots, s_{q-1}$ (*equal* $\vee$ *covers* $\vee$ *contains*) s_q. The latter topological constraints represent an intrinsic property of the spatial hierarchy (i.e., non-intersecting regions). They are defined using well-known topological relationships in two-dimensional spaces (Egenhofer 1991).

This nested spatial hierarchy includes non-intersecting regions at each of its levels. Therefore, topological relationships between regions that respectively represent some events at a same level of the hierarchy are either *disjoint*, *touch* or *equal*. In the context of this paper, we consider the topological relationship *touch* as a non- intersecting spatial relationship, or in other words a weak contact that implies a discontinuity as defined in (Asher and Vieu 1995). Such a topological notion, although not very often used in spatial reasoning, is of particular interest for many physical systems.

3. HIERARCHICAL TEMPORAL REASONING

Let us analyse the relationships between two events $e_1(a_{11}, a_{21}, \ldots, a_{p1})$ and $e_2(a_{12}, a_{22}, \ldots, a_{p2})$ defined using a same calendar $C(A_1, A_2, \ldots, A_p, ..., A_m)$ and a same granularity m, $1 \le p \le m$. The possible hierarchical temporal relationships that apply between these two events are given by the combination of

- Allen's 13 temporal relationships at the granularity level A_p of the these events,
- with the following Allen's 5 temporal relationships (*before*, *after*, *equal*, *meets*, *met_by*) for each successive higher level of the calendar hierarchy. These 5 temporal relationships reflect the nested structure of the calendar hierarchy, that is, temporal relationships between non-intersecting intervals. As applied to the topological relationship *touch* for the spatial dimension, *meets* and *met_by* are considered here as non-intersecting temporal relationships.

Overall, the total number of possible hierarchical temporal relationships, denoted $N^{ht}(p)$, between two events e_1 and e_2, defined at the same level of granularity p of a calendar $C(A_1, A_2, \ldots, A_p, ..., A_m)$ is

$$N^{ht}(p) = 13 \times (5^{p-1})$$

For example, the possible hierarchical temporal relationships for two events represented with a calendar defined with three levels of granularity C_1(*Year*, *Month*, *Day*) are given by the combination of the (*before* $\vee$ *after*, *equal* $\vee$ *meets* $\vee$ *met_by*), (*before* $\vee$ *after* $\vee$ *equal* $\vee$ *meets* $\vee$ *met_by*) and (*before* $\vee$ *after* $\vee$ *equal* $\vee$ *meets* $\vee$ *met_by* $\vee$ *starts* $\vee$ *started* $\vee$ *finishes* $\vee$ *finished* $\vee$ *overlaps* $\vee$ *overlapped* $\vee$ *during* $\vee$ *contains*) disjunction of temporal operations, that is $N^{ht}(3) = 325$ hierarchical temporal relationships.

Let us illustrate the potential of our model with a simplified example. We consider a daily crime event that took place between 10:00 and 11:00 on July the 10^{th} (without a loss of generality, we consider an application that takes place during the same year). Crime investigators want to identify closely related events in the same day, and at previous days (we will not consider the spatial dimension in this section). This event is represented using a calendar C_1(*Month*, *Day*, *Hour*), its temporal value is e_1(July, 10, [10:00,11:00]). Then, events that happen the same year, day and time are evaluated with a (e_1 first operand)

ht(*equal*, *equal*, *equal*) hierarchical temporal operation

Events that happen the same month and day immediately before or after the time of this event are evaluated with a

ht(*equal*, *equal*, *meet* ∨ *met_by*) disjunction of hierarchical temporal operations;

Events that happen the same month, the previous day, and at the same time of this event are evaluated with a

ht(*equal*, *met_by*, *equal*) hierarchical temporal operation;

Events that happen the same month, previous non immediate days, and at the same time of this event are evaluated with a

ht(*equal*, *after*, *equal*) hierarchical temporal operation;

Events that happen the same month, the previous day, and just before of after the time of this event are evaluated with a

ht(*equal*, *met_by*, *meet* ∨ *met_by*) disjunction of hierarchical temporal operations;

Finally, events that happen the same month, previous non immediate days just before or after the time of this event are evaluated with a

ht(*equal*, *after*, *meet* ∨ *met_by*) disjunction of hierarchical temporal operations.

Event patterns can be also analysed. Events that happen the same month are evaluated with a *ht*(*equal, t-any, t-any*) operation, a same day with a *ht*(*t-any, equal, t-any*) operation, at a same time with a *ht*(*t-any, t-any, equal*) operation where *t-any* denotes the possible temporal operations for a considered level of the calendar hierarchy. These examples show the flexibility of such a hierarchical reasoning approach. The complexity of our model is relatively limited as calendars are usually defined with a limited number of levels. This includes, for example, events defined with the finest granularity of a minute or second. For events with a periodic nature at the day level, investigation at the coarser level of month and year becomes redundant or less meaningful. Figure 4 illustrates possible combinations with a calendar, defined with three hierarchical levels (inverse relationships are not represented).

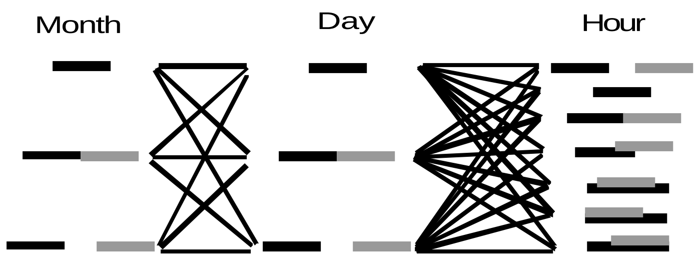

Figure 4: Hierarchical temporal relationships –
Example of a three level calendar

4. HIERARCHICAL SPATIAL AND TEMPORAL REASONING

Each level of a spatial hierarchy has its own set of regions and topological relationships that can be used in spatial reasoning. The underlying constraints of the spatial hierarchy imply that two events located in space, respectively represented by two regions that are *equal* at a level of the spatial hierarchy, are either *equal*, *disjoint* or *touch* at a lower level – if any - of the spatial hierarchy. Similarly, two events located in space, represented by regions that *touch* at a considered level of the spatial hierarchy are either *disjoint* or *touch* at a lower level – if any - of the hierarchy. Finally, two events located in space, *disjoint* at a considered level of the spatial hierarchy, are *disjoint* at a lower level – if any - of the spatial hierarchy (Figure 5). These lead to 3 possible hierarchical relationships for a one-level spatial hierarchy, 6 for a two-level spatial hierarchy, 10 for a three-level hierarchy etc. Overall, the number of possible hierarchical topological relationships, denoted $N^{hs}(q)$, between two events $e_1(a_{11}, a_{21}, \ldots, a_{p1})$ and $e_2(a_{12}, a_{22}, \ldots, a_{p2})$, defined at the same level of granularity p of a calendar $C(A_1, A_2, \ldots, A_p, \ldots, A_m)$, respectively located in e_{s1} $(s_{11}, s_{21}, \ldots, s_{q1})$ and e_{s2} $(s_{12}, s_{22}, \ldots, s_{q2})$ at the same level q of abstraction of a spatial hierarchy $SH(S_1, S_2, \ldots, S_q, \ldots, S_n)$, is

$$N^{hs}(q) = 1 + 2 + \ldots + q + (q + 1)$$

We can remark that these hierarchical topological relationships have some singularities if compared to the previously defined hierarchical temporal relationships. The main differences come from the fact that time is oriented whereas space is not; then two additional inverses are defined in time, that is, *met_by* and *after* for the temporal relationships *meets* and *before*, respectively. Moreover space is not cyclic by nature although time is (i.e., the assumptions of our model). We have also restricted our model to non-intersecting regions at all levels of the spatial hierarchy. Let us remark that a relaxed constraint at the lower level of the spatial hierarchy, that is, application of the complete set of 8 topological relationships as defined in (Egenhofer, 1991) will lead to an increased number of potential topological relationships, that is, $8 \times (5^{q-1})$.

In accordance with our hierarchical approach to temporal reasoning, let us assume that the crime event introduced in Section 3 can be located at either the building, neighbourhood or district spatial hierarchy levels. Crime investigators at the neighbourhood level may be interested in those events which happened in the same neighbourhood, whilst investigators working at district level may be interested in those events which happened in the same district. In the spatial dimension, our example event is represented using a spatial hierarchy, its value is $SH_1(district_1, neighb_1, building_1)$. Therefore, events that happen within the same district, neighbourhood, and building are evaluated with a

hs(*equal, equal, equal*) hierarchical topological operation;

Events that happen within the same district and neighbourhood, but in spatially disjoint buildings are evaluated with a

hs(*equal, equal, disjoint*) hierarchical topological operation;

Events that happen within the same district, but in disjoint neighbourhoods are evaluated with a

hs(*equal, disjoint, disjoint*) hierarchical topological operation;

Events that happen within the same district and touching neighbourhoods are evaluated with a

hs(*equal, touch, disjoint Ú touch*) disjunction of hierarchical topological operations.

Similarly, Figure 5 introduces topological relationships within a three-level spatial hierarchy in an urban context, say, SH_1(*district*, *neighbourhood*, *building*).

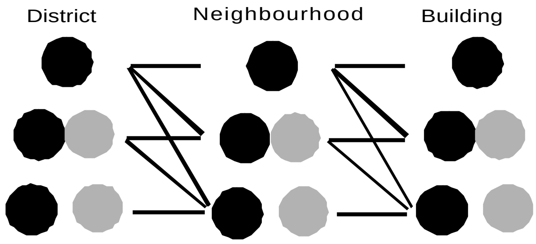

Figure 5: Hierarchical spatial relationships – Example of a three level hierarchy

Hierarchical relationships have been identified in the temporal and spatial dimensions. Hierarchical relationships can be combined for events located in space. A general rule can be formulated as follows under the constraints of our model defined in the temporal and spatial dimensions:

- Let us consider two events $e_1(a_{11}, a_{21}, \ldots, a_{p1})$ and $e_2(a_{12}, a_{22}, \ldots, a_{p2})$, defined at the same level of granularity p of a calendar $C(A_1, A_2, \ldots, A_p, \ldots, A_m)$, respectively located in e_{s1} $(s_{11}, s_{21}, \ldots, s_{q1})$ and e_{s2} $(s_{12}, s_{22}, \ldots, s_{q2})$ at the same level q of abstraction of a spatial hierarchy $SH(S_1, S_2, \ldots, S_q, \ldots, S_n)$. The number of possible hierarchical relationships, denoted $N^{h}(p,q)$, between the events e_1 and e_2 is given by the product of $N^{ht}(p)$ by $N^{hs}(q)$, that is

$$N^{h}(p,q) = (13 \text{ x } (5^{p-1})) \times (1 + 2 + \ldots + q + (q+1))$$

Spatio-temporal queries can be then expressed by the conjunction of hierarchical topological and temporal expressions. For example events that happen the same month, previous non immediate days just before or after the time of a crime; in the same district and neighbourhood, but at topologically *disjoint* buildings as a considered crime event are evaluated with a

ht(*equal, after*, *meet* ∨ *met_by*) ∧ *hs*(*equal, equal, disjoint*) expression;

Events that happen the same month, the previous day at the same time, in the same district and same or "touching" neighbourhoods as a considered crime event are evaluated with a

ht(*equal, met_by, equal*) ∧ *hs*(*equal, touchÚ equal, s-any*) expression.

As topological and temporal hierarchical operations are orthogonal (i.e., defined independently), previously defined topological and temporal hierarchical queries can be combined in either manner. The proposed language uses a set of well known operations in both temporal and spatial dimensions, so its understanding and appropriation by final users is quite a straightforward task. The large number of resulting combined operations offers a powerful set of operations to support the analysis of patterns in either the temporal, spatial or spatio-temporal domain. Moreover, clustered patterns in space and/or time and periodic events can be studied. Such spatio-temporal reasoning capabilities are particularly adapted to explorative analysis developed in crime or epidemiological studies.

It is generally considered that passing and browsing through distinct levels of abstraction imply the composition of represented events (Badaloni and Berati 1994). In particular, real-world events represented at a temporal level of abstraction can only be described at a finer temporal granularity if we also shift to a finer temporal level in order to observe the changes they may produce in the environment (Mota and Robertson 1996). Accordingly, we may consider, in the context of our example, that the analysis of crime events at the city level corresponds to a temporal level of a month, the district level to a temporal level of a day, and neighbourhood and building levels to a temporal level of an hour. Therefore, the analysis of topological and temporal relationships can be represented in a co-ordinated way. For example, one may consider that the city level of the spatial hierarchy can be associated to a temporal granularity of a month, the district and neighbourhood spatial levels of abstraction to a temporal granularity of a day, and finally the building spatial level to a temporal granularity of an hour; obviously, these assumptions are application dependent.

5. CONCLUSION

The analysis of spatio-temporal phenomena in GIS often implies reasoning at different temporal and spatial levels of abstraction. The research described in this paper introduces an integrated approach to the development of hierarchical reasoning in time and space. The model is based on the manipulation of events represented as hierarchical data types modelled using the concept of calendar that reflects the cyclic nature of time. Resulting temporal operations are then identified as the possible combinations of temporal operations at each level of the temporal hierarchy. By extension, we apply similar hierarchical concepts to the spatial dimension. This leads to a multi-scale and hierarchical representation of event relationships in space and time. Overall, the model supports the identification of relationships in time and space at different hierarchical levels. The model is flexible enough to support application orientated relationships in both temporal and spatial dimensions. We have illustrated this flexibility with events "contained" within a period unit defined at the immediate superior level of their calendar hierarchy and located in non-intersecting regions. Similar – or relaxed - constraints can be defined according to application dependent strategies.

This hierarchical approach of relationships in space and time offers a flexible algebra for the manipulation of geo-referenced events. As our model is based on well-known relationships in time and space, its computational complexity is relatively limited, and its implementation is feasible. It is flexible enough to apply in different application contexts such as crime studies, urban traffic monitoring or epidemiological studies. Current work concerns the computational implementation of these hierarchical relationships and the extension of the model to events defined with intersecting space-time hierarchical units.

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